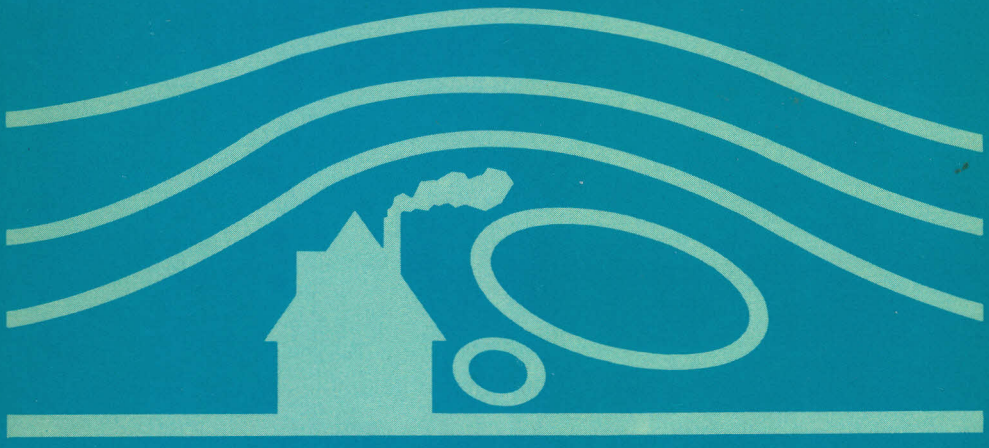
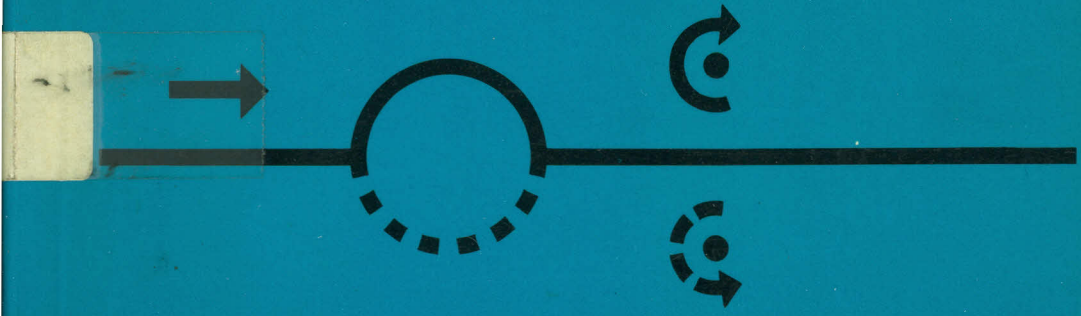
A first course in fluid dynamics

A.R. PATERSON





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fluid dynamics

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